南開大學
Nankai University

2022 天津图论前沿研讨会

秩序册

南开大学数学交叉科学中心

图论与组合优化团队

南开大学数学交叉科学中心

中国-天津

南開大學
Nankai University

2022 天津图论前沿研讨会

（线上）

会议时间：2022 年 11 月 5 日-6 日（周六日）

腾讯会议：880 7369 0483（密码：1105）

会议举办方

南开大学数学交叉科学中心

图论与组合优化团队

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会议日程

11 月 5 日	下　午	腾讯会议：880 7369 0483 （密码：1105）
14:20–14:30	开幕式	
主持人	**宁　博**	
14:30-15:00	蔡俊青	Spanning trees with bounded number of leaves and branches
15:00-15:30	胡　杰	Graph tilings in incompatibility systems
15:30–15:40	茶　歇	
主持人	**岳　军**	
15:40-16:10	Quaid Iqbal	Distance Regular Graphs with diameter $D = 3$ with fixed eigenvalue $a_2 - c_3$, under certain conditions
16:10-16:40	冀　蒙	Remarks on (CO-)irredundant and mixed Ramsey number
16:40-17:10	房宜宾	Characterizing forbidden pairs for relative length of longest paths and cycles
11 月 6 日上午　自由讨论		
11 月 6 日	下　午	腾讯会议：880 7369 0483 （密码：1105）
主持人	**张俊雪**	
13:30-14:00	王周宁馨	Circular Flow in Eulerian signed graphs
14:00-14:25	李路易	Rainbow structures in a collection of graphs with degree conditions
14:25-14:50	苏　博	Some Results about the 5-Flow Conjecture
14:50-15:15	王志谦	On Circular Chromatic Number of Signed Planar Graph of Girth At Least 5
15:15–15:30	茶　歇	

主持人	李路易	
15:30-15:55	韦春燕	Homomorphisms of planar graphs
15:55-16:20	郗常青	The k-component domination numbers in graphs
16:20-16:45	杨　宁	Unified approach for spectral properties of weighted adjacency matrices for graphs with degree-based edge-weights
16:45-17:10	张俊雪	Extremal numbers for vertex disjoint copies of cliques

Spanning trees with bounded number of leaves and branches

蔡俊青（天津师范大学）

Abstract： A natural extension of the hamiltonian path problem is to look for conditions that guarantee the existence of a spanning tree with few leaves or few branch vertices. Many researchers have investigated degree sum conditions and forbidden subgraph conditions for the existence of spanning trees with few leaves, few branch vertices, or few sum of branch vertices and leaves. In this report, we will give results about spanning tree problems.

Graph tilings in incompatibility systems

胡杰（南开大学）

Abstract： Given two graphs H and G, an H-tiling of G is a collection of vertex-disjoint copies of H in G and an H-factor is an H-tiling that covers all vertices of G. Kuhn and Osthus managed to characterize, up to an additive constant, the minimum degree threshold which forces an H-factor in a host graph G. We study a similar tiling problem in incompatibility systems. The notion of incompatibility system was first proposed by Krivelevich, Lee and Sudakov to study the robustness of Hamiltonicity of Dirac graphs. We derive a best possible minimum degree condition for H-factors in incompatibility systems, which can be regarded as a robust version of the Kuhn–Osthus result. This is a joint work with Hao Li, Yue Wang and Donglei Yang.

Distance Regular Graphs with diameter $D = 3$ with fixed eigenvalue $a_2 - c_3$, under certain conditions.

Quaid Iqbal （南开大学）

Abstract： Let Γ be a non-bipartite distance-regular graph with diameter 3 and distinct eigenvalues $k = \theta_0 > \theta_1 > \theta_2 > \theta_3$, then the distance-2 graph Γ_2 of Γ is strongly regular, if, and only if, $a_2 - c_3$ is an eigenvalue of Γ. In this talk, we will give several kinds of classifications of non-bipartite distance-regular graphs with diameter 3 and eigenvalue $a_2 - c_3$ under various conditions, for example the valency k of Γ is at most $2(a_1 + 1)$, $c_3 \leq 9$, $a_2 \leq 7$, $\tilde{c} \leq 26$ (where $\tilde{c}$ is the number of common neighbors between any two non-adjacent vertices) and $\theta_{min}(\Gamma) > -3$.
This is a joint work with Prof. J.H. Koolen, Prof. Jongyook Park and Dr. Masood Ur Rehman.

Remarks on (CO-)irredundant and mixed Ramsey number

冀蒙（天津师范大学）

Abstract： A set of vertices $X \subseteq V$ in a simple graph $G(V, E)$ is irredundant if each vertex $x \in X$ is either isolated in the induced subgraph $\langle X \rangle$ or else has a private neighbor $y \in V \setminus X$ that is adjacent to x and to no other vertex of X. a vertex subset $X \subseteq V$ is called CO-irredundant if every vertex $v \in X$ either contains no neighbors in X or else has a private neighbor $y \in V$ that is adjacent to x and to no other vertex of X. The irredundant Ramsey number $s(m, n)$ is the smallest N such that in every red-blue coloring of the edges of the complete graph of order N, either the blue subgraph contains an m-element irredundant set or the red subgraph contains an n-element irredundant set. The mixed Ramsey number $t(m, n)$ is the smallest N for which every red-blue coloring of the edges of K_N yields an m-element irredundant set in the blue subgraph or an n-element independent set in the red subgraph. The CO-irredundant Ramsey number $s_{CO}(m, n)$ is the smallest N such that any red-blue edge coloring of K_N yields either an m-element CO-irredundant set in the blue graph or an n-element CO-irredundant set in the red graph. In this report, we first improve the upper bound of $t(3, n)$; using this result, we confirm that a conjecture proposed by Chen, Hattingh, and Rousseau, that is, $\lim_{n\to\infty} \frac{t(m,n)}{r(m,n)} = 0$ for each fixed $m \leq 4$. Furthermore, we establish an asymptotic lower bound for CO-irredundant Ramsey number of K_N.

Characterizing forbidden pairs for relative length of longest paths and cycles

房宜宾（南开大学）

Abstract： The relative length of a graph is the difference of the order of a longest path and a longest cycle. In this talk, we present a complete characterization of the forbidden pairs H such that every 2-connected H-free graph has relative length at most one. We also investigate the forbidden pairs needed to guarantee a 2-connected H-free graph has constant bounded relative length

Circular Flow in Eulerian signed graphs

王周宁馨（南开大学）

Abstract： In this talk, we consider analogs of Jaeger' s circular flow conjecture and its dual Jaeger-Zhang conjecture in signed graphs. We will first give the notions of circular coloring and circular flow in signed graphs, and then show that every $(6k-2)$-edge-connected Eulerian signed graph admits a circular $\frac{4k}{2k-1}$-flow and that every signed bipartite planar graph of negative-girth at least $(6k-2)$ admits a circular $\frac{4k}{2k-1}$-coloring (equivalently, admits a homomorphism to a negative cycle of length $2k$). This is based on joint work with Jiaao Li, Reza Naserasr, and Xuding Zhu.

Rainbow structures in a collection of graphs with degree conditions

李路易（南开大学）

Abstract: Let $\mathbf{G} = \{G_1, \ldots, G_s\}$ be a collection of not necessarily distinct graphs on the same vertex set V. From another perspective, we can see $\mathbf{G}$ as an edge-colored multigraph $\widetilde{\mathbf{G}}$ with $V(\widetilde{\mathbf{G}}) = V$ and $E(\widetilde{\mathbf{G}})$ a multiset consisting of $E(G_1), \ldots, E(G_s)$, and the edge e of $\widetilde{\mathbf{G}}$ is colored by i if $e \in E(G_i)$. A graph is *rainbow* in $\mathbf{G}$ if any two edges belong to different graphs of $\mathbf{G}$. We say that $\mathbf{G}$ is *rainbow vertex-pancyclic* if each vertex of V is contained in a rainbow cycle of length ℓ for every integer ℓ with $3 \leq \ell \leq n$, and $\mathbf{G}$ is *rainbow panconnected* if for any pair of vertices u and v of V there exists a rainbow path of length ℓ connecting u and v for every integer $\ell \in [d_{\widetilde{\mathbf{G}}}(u, v), n-1]$. In this paper, we study the existences of rainbow spanning trees and rainbow Hamiltonian paths in $\mathbf{G}$ under some degree-sum conditions. Then, we study the rainbow vertex-pancyclicity, rainbow panconnectedness and the existence of rainbow cliques in $\mathbf{G}$ under some degree conditions. We also give some examples to show the sharpness of our results. This is a joint work with Ping Li and Xueliang Li.

Some Results about the 5-Flow Conjecture

苏博（南开大学）

Abstract: Tutte in 1954 introduced the integer flow theory in order to deal with a series of problems in graph theory, and he proposed the classic 5-flow conjecture that every bridgeless graph admits a nowhere-zero 5-flow. This conjecture has been widely studied. Many scholars have also verified the existence of nowhere-zero 5-flows for some special classes of graphs, but the 5-flow conjecture itself is still open, which has been neither proved nor disproved so far. In this lecture, we mainly summarize some properties of the potential minimal counterexamples to the 5-flow conjecture, which are expected to provide a glimpse of the possibility that the 5-flow conjecture holds or has counterexamples.

On Circular Chromatic Number of Signed Planar Graph of Girth At Least 5

王志谦（南开大学）

Abstract： In 2020, the concept of a circular r-coloring of a signed graph (G, σ) was introduced by X. Zhu et, which is the extension of circular r-coloring brought up by Vince in 1988. Recently, J. Li et introduced the concept of circular r-flow as the dual notion of circular r-coloring in the sense of signed planar graphs, and several results of edge connectivity and circular flow index (equivalently, girth and circular chromatic number) was obtained. X. Zhu et also gave a tight upper bound of circular chromatic number of signed planar bipartite graphs in 2022. In this paper, we continue on their work and proved that for every signed planar graph of girth at least 5 with n vertices, the circular chromatic number is no more than $4 - \frac{12}{5n-4}$. At the end, we proposed two conjectures about this upper bound.

Homomorphisms of planar graphs

韦春燕（南开大学）

Abstract： A graph G is said to be strongly $\mathbb{Z}_\ell$-connected if for any mapping $\theta : V(G) \to \mathbb{Z}_\ell$ satisfying that $\sum_{v \in V(G)} \theta(v) \equiv |E(G)| \pmod{\ell}$, there is an orientation D on G such that $d_D^+(v) \equiv \theta(v) \pmod{\ell}$ for each $v \in V(G)$ where $d_D^+(v)$ is the out-degree of v in D. In this paper, we prove that every $(6k-4)$-edge-connected planar graph is strongly $\mathbb{Z}_{2k}$-connected for $k \in \{2, 3, 4\}$, which improves a general result of Lovász, Thomassen, Wu, and Zhang [J. Combin. Theory Ser. B, 103 (2013) 587–598] for those cases. Furthermore, we have two applications of this result on homomorphisms and circular colorings of signed graphs. We first show that every signed bipartite planar graph of negative-girth at least $6k-4$ admits a homomorphism to the negative even cycle C_{-2k} for $k \in \{2, 3, 4\}$, where the $k = 2$ case was previously obtained in [J. Combin. Theory Ser. B, 153 (2022) 81–104] through different approaches and the $k = 3, 4$ cases improve several known results. The second corollary provides some upper bounds on the circular chromatic numbers of signed planar graphs (G, σ) of given girth g: if $g \geqslant 7$, then (G, σ) is circular 3-colorable, and if $g \geqslant 10$, then (G, σ) is circular $\frac{8}{3}$-colorable.

The k-component domination numbers in graphs

郗常清（南开大学）

Abstract: Let k be a positive integer and let $G = (V(G), E(G))$ be a graph. A vertex set D is a k-component dominating set of G if every vertex outside D in G has a neighbor in D and every component of the subgraph $G[D]$ of G induced by D contains at least k vertices. The minimum cardinality of a k-component dominating set of G is the *k-component domination number* $\gamma_k(G)$ of G. It was conjectured that if G is a connected graph of order $n \geq k+1$, and minimum degree at least 2, then $\gamma_k(G) \leq \frac{2kn}{2k+3}$ except a finite set of graphs. In this paper, we focus on the parameter $\gamma_3(G)$ of G. We first determine the exact values of 3-component domination number of paths and cycles. And then, we get the following result: For every connected graph G of order n with minimum degree at least 2 and maximum degree at most 3, then $\gamma_3(G) \leq \frac{2n}{3}$ except seven special graphs. This result provides positive support for the conjecture and also generalizes a result of Alvarado et al. [Discrete Math., 2016]. Moreover, we study the k-component domination number on the generalized Petersen graph $P(n, m)$ and show the exact values of 3-component domination numbers on $P(n, 1)$ and $P(n, 2)$.

Unified approach for spectral properties of weighted adjacency matrices for graphs with degree-based edge-weights

杨宁（南开大学）

Abstract: Let G be a graph and d_i denote the degree of a vertex v_i in G, and let $f(x, y)$ be a real symmetric function. Then one can get an edge-weighted graph in such a way that for each edge v_iv_j of G, the weight of v_iv_j is assigned by $f(d_i, d_j)$. Hence, we have a weighted adjacency matrix $A_f(G)$ of G, in which the ij-entry is equal to $f(d_i, d_j)$ if $v_iv_j \in E(G)$ and 0 otherwise. In this talk we will use a unified approach to deal with the spectral properties of $A_f(G)$ for $f(x, y)$ to be the functions of graphical or topological function-indices. Firstly, we study the interlacing inequalities under some kinds of graphs transformations including edge deletion, edge subdivision, vertex deletion and vertex contraction. Secondly, we establish a uniform equivalent condition for a connected graph G to have m distinct weighted adjacency eigenvalues.

Extremal numbers for vertex disjoint copies of cliques

张俊雪（南开大学）

Abstract： The Turán number $ex(n, H)$ of H is the maximum number of edges of an n-vertex simple graph containing no copy of H as a subgraph. Denote $EX(n, H)$ as the set of graphs that have no copy of H as a subgraph and with size $ex(n, H)$. In this paper, utilizing a celebrated theorem of Hajnal and Szemerédi together with some results of Chen, Lih, and Wu, and of Kierstead and Kostochka, we determine $ex(n, 3K_{p+1})$ and $ex(t(p+1), tK_{p+1})$, and characterize all extremal graphs $EX(t(p+1), tK_{p+1})$ for any positive integers t, n, and p with $p \geq 2$.