南開大學
Nankai University

2022 年组合学前沿研讨会

会议手册

南开大学数学交叉科学中心

“组合数学”创新团队

2022 年 10 月

南開大學
Nankai University

会议主题：为促进国内组合学的发展，本次论坛将邀请国内组合学者交流本领域最新学术成果及进展。研究前沿问题，探讨未来发展，增进国内学术交流与合作。

会议形式：线上会议

- 会议时间：2022 年 10 月 8 日-10 日
- 线上会议地址：腾讯会议号：944-5632-3400　(密码：202210)

主办单位：南开大学数学交叉科学中心

“组合数学”创新团队

校内会务组（按姓名首字母排序）：

谷珊珊、郭龙、金应烈、路在平、孙慧、王博、王星炜、吴腾、杨立波

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组合学前沿研讨会日程

日期	时 间	报告人	报告题目
10月8日	**腾讯会议：944-5632-3400　　密码：202210**		
	08:15-08:20	开幕式	
	主持人：杨立波		
	08:20-09:00	冯荣权	Two classes of power mappings with boomerang uniformity 2
	09:00-09:40	王　毅	Analytic aspects of (generalized) central trinomial coefficients
	09:40-10:20	张文静	Asymptotic formulas in theory of partitions
	10:20-10:30	休　息	
	主持人：谷珊珊		
	10:30-11:10	范久瑜	Poincaré polynomials of odd diagram classes
	11:10-11:50	周　悦	On the number of inequivalent almost perfect nonlinear functions
	11:50-14:00	午　休	
	主持人：路在平		
	14:00-14:40	杜少飞	Two results related to finite field theory
	14:40-15:20	王恺顺	Erdős-Ko-Rado theorem, Hilton-Milner theorem and related results
	15:20-16:00	周进鑫	Locally bi-2-transitive graphs and cycle-regular graphs
	16:00-16:10	休　息	
	主持人：郭龙		
	16:10-16:50	熊　欢	Some applications of hyperplane arrangements in deep learning
	16:50-17:30	苏长剑	Alcove walks and Chevalley formula for the motivic Chern classes

日期	时 间	报告人	报告题目
10月9日	**腾讯会议：944-5632-3400　　密码：202210**		
	主持人：王星炜		
	10:00-10:40	夏先伟	New equalities and inequalities for the ranks and cranks of partitions
	10:40-11:20	王国亮	Recent progress on the e-positivity of spiders
	11:20-14:00	午　休	
	主持人：孙慧		
	14:00-14:40	祝宝宣	Zeros of combinatorial polynomials and some related properties
	14:40-15:20	王六权	Rogers-Ramanujan type identities
	15:20-16:00	贺　兵	An extensions of some expansion formulas of Liu
	16:00-16:10	休　息	
	主持人：张彪		
	16:10-16:50	傅士硕	Burstein's permutation conjecture, Hong and Li's inversion sequence conjecture, and restricted Eulerian distributions
	16:50-17:30	王岁杰	Classifications on hyperplane arrangements
日期	**时 间**	**报告人**	**报告题目**
10月10日	**腾讯会议：944-5632-3400　　密码：202210**		
	主持人：杨立波		
	08:20-09:00	冯　涛	On the interplay between group theory and finite geometry
	09:00-09:40	周胜林	On flag-transitive 2-designs
	09:40-10:20	林志聪	Combinatorics of integer partitions with prescribed perimeter

报告摘要

Two Results Related to Finite Field Theory

杜少飞　首都师范大学

During our recent researches, two problems are involved, which are related to finite field theory and number theory as well:

Problem 1: given a monic polynomial $f(x)$ of degree n over the finite field F_q, find a primitive element β such that $f(\beta) \in F_q^2$;

Problem 2: determine and enumerate all companions matrices M matrix over F_q such that $M^d = -I$ for some integer d, where I is the identity matrix.

In this talk, we shall give an answer to Problem 1 when $n \leq 8$ and to Problem 2, respectively.

Poincaré Polynomials of Odd Diagram Classes

范久瑜　四川大学

An odd diagram class is a set of permutations with the same odd diagram. Brenti, Carnevale and Tenner showed that each odd diagram class is an interval in the Bruhat order. They conjectured that such intervals are rank-symmetric. In this talk, we present an algorithm to partition an odd diagram class in a uniform manner. As an application, we obtain that the Poincaré polynomial of an odd diagram class factors into polynomials of the form $1+t+\cdots+t^m$. This in particular resolves the conjecture of Brenti, Carnevale and Tenner. This talk is based on joint work with Peter L. Guo.

On the interplay between group theory and finite geometry

冯涛　浙江大学

Jacques Tits introduced the notion of buildings to serve as a bridge between simple groups of Lie types and geometry. In Aschbacher's classification of subgroups of classical groups, finite geometric structures play an important role. Classical groups arise as automorphism groups of geometric structures, and the most interesting geometric structures are those with high degree of symmetries. There has been extensive work on geometric structures that satisfy a certain transitivity hypothesis, and it remains an active research area. In this talk, I will briefly describe this interplay between group theory and finite geometry and report some recent progress on the classification of finite geometric structures.

Burstein's permutation conjecture, Hong and Li's inversion sequence conjecture, and restricted Eulerian distributions

傅士硕　重庆大学

In this talk, we address two conjectures on the enumeration of pattern restricted permutations (resp. inversion sequences) due to Burstein (resp. Hong and Li). Indeed, we will not only confirm Hong and Li's conjecture and Burstein's first conjecture, but also establish two more delicate generating function identities with the ides (for inverse descent) statistic concerned in the restricted permutation case, and the asc statistic concerned in the restricted inversion sequence case, which yield a new equidistribution result. The talk is based on joint work with Shane Chern and Zhicong Lin.

A_n extensions of some expansion formulas of Liu

贺兵　中南大学

In this talk, we extend certain expansion formulas of Liu to multiple basic hypergeometric series over the root system A_n. The usefulness of Liu's formulas in special functions and number theory has been shown by Liu and many others. We first establish a very general multiple expansion formula over the root system A_n and then deduce several A_n extensions of a well-known expansion formula of Liu. From these A_n extensions of Liu's expansion formula, we derive some multiple expansion formulas for infinite products. Some applications are also given.

Combinatorics of integer partitions with prescribed perimeter

林志聪　山东大学数学与交叉科学研究中心

We prove that the number of even parts and the number of times that parts are repeated have the same distribution over integer partitions with fixed perimeter. This refines Straub's analogue of Euler's Odd-Distinct partition theorem. We generalize the two concerned statistics to these of the part-difference less than d and the parts not congruent to 1 modulo $d+1$ and prove a distribution inequality, that has similar flavor as Alder's ex-conjecture, over partitions with prescribed perimeter. Both of our results are proved analytically and combinatorially. This talk is based on joint work with Huan Xiong and Sherry H.F. Yan.

Alcove walks and Chevalley formula for the motivic Chern classes

苏长剑　清华大学

I will talk about an ongoing joint work with L. Mihalcea and H. Naruse. We use the alcove walk model of A. Ram to give a Chevalley formula for the motivic Chern classes of the Schubert cells in the flag varieties. Applications of this include computations of the K-theoretic stable envelopes and the Iwahori–Whittaker functions for a p-adic group.

Recent progress on the e-positivity of spiders

王国亮　北京理工大学

Motivated by Stanley and Stembridge's 3+1 conjecture on the e-positivity of chromatic symmetric functions of certain graphs, and by Dahlberg, She, and van Willigenburg's conjecture on the e-positivity of certain trees, we present recent progress on the e-positivity of spiders with at most 5 legs. We proved that e-positive spiders with 5 legs has at least 405 vertices

Erdős-Ko-Rado theorem, Hilton-Milner theorem and related results

王恺顺　北京师范大学

Erdős-Ko-Rado theorem and Hilton-Milner theorem are two of the fundamental results in combinatorics, which are closely related to graph theory, association schemes, codes, designs et al. Over the last decades, they have been extended to many classical objects. In this talk, I will introduce the two theorems and their generalization.

Rogers-Ramanujan type identities

王六权　武汉大学

Recently, Rosengren utilized an integral method to prove a number of conjectural identities found by Kanade and Russell. Using this integral method, we give new proofs to some double sum identities of Rogers-Ramanujan type. Our proofs are based on streamlined calculations, which relate these double sum identities to some known Rogers-Ramanujan type identities with single sums. Moreover, we prove a conjectural identity of Andrews and Uncu which was earlier confirmed by Chern. Using the integral method, we also establish a number of new Rogers-Ramanujan type identities involving double and triple sums. Part of this talk is based on a joint work with Zhineng Cao (曹智能).

Classifications on Hyperplane Arrangements

王岁杰　湖南大学

In this talk, I will introduce several combinatorial classifications on hyperplane arrangements, including parallel translations, one element extensions, one-dimensional extensions, and k-dimensional restrictions.

Analytic aspects of (generalized) central trinomial coefficients

王毅　大连理工大学

The divisibility and congruence properties of the usual and generalized central trinomial coefficients have been extensively studied. In this talk we consider analytic properties of these numbers.

New equalities and inequalities for the ranks and cranks of partitions

夏先伟　苏州科技大学

Let $p(n)$, $N(r,m;n)$ and $C(r,m;n)$ denote the number of partitions of n, the number of partitions of n with rank congruent to r modulo m and the number of partitions of n with crank congruent to r modulo m, respectively. Applying some properties of Appell-Lerch sums and a universal mock theta function $g(x,q)$, we establish the generating functions for $N(a,12;n)$ and $C(a,12;n)$ with $0 \leq a \leq 11$. With those generating functions, we obtain some new equalities and inequalities involving $p(n)$, $N(a,12;n)$ and $C(a,12;n)$. In particular, we confirm several conjectures due to Aygin and Chan and prove that for $n \geq 0$,

$$N(2,12;2n)+N(5,12;2n)+N(6,12;2n)=C(1,12;2n)+C(3,12;2n)+C(5,12;2n).$$

This work was jointed with Fan and Zhao.

Some Applications of Hyperplane Arrangements in Deep Learning

熊欢　哈尔滨工业大学

Over the past decade, deep Neural Networks (NNs), especially deep Convolutional Neural Networks (CNNs), have attracted much attention and achieved state-of-the-art results in many machine learning tasks. An explanation for the superiority of NNs is that they can realize a large family of complicated functions, i.e., they have powerful expressivity. The expressivity of a Neural Network with Piecewise Linear activations (PLNN) can be quantified by the maximal number of linear regions it can separate its input space into. In this talk, we build some connections between hyperplane arrangements and Piecewise Linear Convolutional Neural Networks (PLCNNs), and use them to derive the maximal and average numbers of linear regions for one-layer PLCNNs. Furthermore, we obtain upper and lower bounds for the number of linear regions of multi-layer PLCNNs. Rectified Linear Unit (ReLU) is a piecewise linear activation function that has

been widely adopted in various architectures. Our results suggest that deeper ReLU CNNs have more powerful expressivity than their shallow counterparts, while ReLU CNNs have more expressivity than fully-connected ReLU NNs per parameter, in terms of the number of linear regions.

Asymptotic formulas in theory of partitions

张文静　湖南大学

Let $\overline{N}_2(a,c,n)$ be the number of overpartitions of n whose the M_2-rank is congruent to a modulo c and $M_k(a,c,n)$ be the number of k-colored partitions of n whose the k-crank is congruent to a modulo c. In this talk, we introduce the asymptotic formula of $\overline{N}_2(a,c,n)$ and $M_k(a,c,n)$ utilizing the Ingham Tauberian Theorem and Hardy-Ramanujan circle method, respectively. As applications, we derive inequalities concerning with $\overline{N}_2(a,c,n)$ including its strict log-subadditivity and log-concavity. This work is joint with Ying Zhong.

Locally bi-2-transitive graphs and cycle-regular graphs

周进鑫　北京交通大学

A vertex-transitive but not edge-transitive graph G is called *locally bi-2-transitive* if the stabiliser S in the full automorphism group of G of every vertex v of G has two orbits of equal size on the neighbourhood of v, and S acts 2-transitively on each of these two orbits. Also a graph is called *cycle-regular* if the number of cycles of a given length passing through a given edge in the graph is a constant, and a graph with girth g is called *edge-girth-regular* if the number of cycles of length g passing through any edge in the graph is a constant.

In this talk, I shall discuss the characterization of edge-girth-regular and locally bi-2-transitive graphs of girth 3. It is proved that a graph of girth 3 is edge-girth-regular and locally bi-2-transitive if and only if G is the line graph of a semi-symmetric locally 3-transitive graph. Then as an application, we prove that every tetravalent edge-girth-regular locally bi-2-transitive graph of girth 3 is cycle-regular. This shows that

vertex-transitive cycle-regular graphs need not to be edge-transitive, and hence resolves the problem posed by Fouquet and Hahn at the end of their paper 'Cycle regular graphs need not be transitive', in *Discrete Appl. Math.* 113 (2001) 261–264.

On flag-transitive 2-designs

周胜林　华南理工大学

A 2-(v, k, λ) design $\mathcal{D}$ is a pair $(P,\ \mathcal{B})$ where P is a v-set(the elements are called points) and $\mathcal{B}$ is a collection of k-subsets of P(its elements called blocks) such that each 2-subset of P is contained in exactly λ blocks. The full automorphism group $Aut(\mathcal{D})$ of $\mathcal{D}$ consists of all permutations of P that leave $\mathcal{B}$ invariant. The group $G \leq Aut(\mathcal{D})$ is called flag-transitive if G cts transitively on the set of flags of $\mathcal{D}$, where a flag is a point-block pair (α, B) such that $\alpha \in B$. In this talk, I will introduce some new reduction theorems for flag-transitive 2-(v, k, λ) designs under one of the following additional conditions: (i) $(r-\lambda, k) = 1$; (ii) $r > \lambda(k-3)$; (iii) $(v-1, k-1) \leq (v-1)^{1/2}$. Some related classification results are also given. This is a joint work with Dr. Wanbao Zhang, Yanwei Zhao and Chuyi Zhong.

On the number of inequivalent almost perfect nonlinear functions

周悦　国防科技大学理学院数学系

A function $f : \mathbb{F}_{2^n} \to \mathbb{F}_{2^n}$ is called *almost perfect nonlinear* (APN, for short), if the map

$$x \mapsto f(x + a) - f(x),$$

is 2-to-1 for each nonzero $a \in \mathbb{F}_{2^n}$.

APN functions play an important role in the design of block ciphers as they offer the strongest resistance against differential cryptanalysis. Despite more than 25 years of research, only a limited number of APN functions are known.

In this talk, we show that a construction by Taniguchi provides at least $\frac{\varphi(m)}{2}\left\lceil\frac{2^m+1}{3m}\right\rceil$ inequivalent APN functions on $\mathbb{F}_{2^{2m}}$, where φ denotes Euler's totient function. This is

a great improvement of previous results: for even m, the best known lower bound has been $\frac{\varphi(m)}{2}\left(\lfloor\frac{m}{4}\rfloor+1\right)$, for odd m, there has been no such lower bound at all. Moreover, we also present several recent construction results and some open questions.

Zeros of combinatorial polynomials and some related properties

祝宝宣　江苏师范大学

Zeros of polynomials play an important role in mathemaics. In this talk, we will report some results for zeros of combinatorial polynomials and some applications.